disequazioni Fratte con termini di 1° e 2° grado

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Disequazioni Fratte

Sono disequazioni del tipo :

$$\frac{N}{D} > 0$$

Vediamo come procedere per risolverle.

Risoluzione

Alle soluzioni si può giungere utilizzando il seguente metodo pratico:

1. **Poniamo sempre e comunque**

   il numeratore N >0

   il denominatore N >0

2. **Risolvo singolarmente le disequazioni ottenendo le soluzioni**

   S1 della prima disequazione

   S2 della seconda disequazione

3. **Rappresentiamo sulla retta orientata, a livelli distinti, le soluzioni S1 ed S2 .**

4. **Ricavo dal grafico le soluzioni della disequazione fratta.**

   Saranno valori concordi se sto risolvendo una disequazione Fratta > 0

   Saranno valori discordi se sto risolvendo una disequazione Fratta < 0

Esempio 1

$$\frac{x^2+2}{x} < 0$$

1) **Pongo numeratore** $x^2 + 2 > 0$

Pongo denominatore, $x > 0$

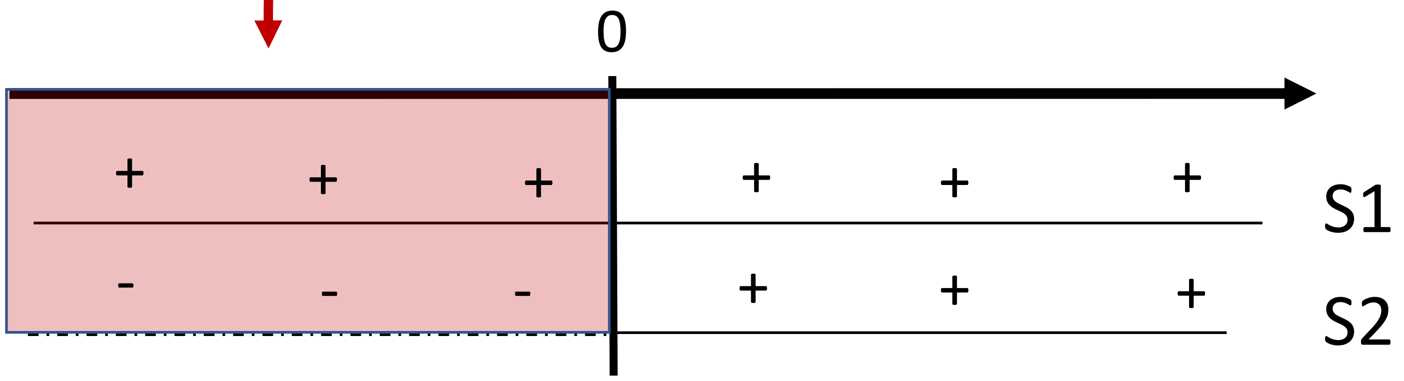
2) **Risolvo singolarmente :**

$N > 0.$ $\quad x^2 + 2 > 0 \quad$ S1: $\forall x \in R$

$D > 0.$ $\quad x > 0. \quad$ S2: $x > 0$

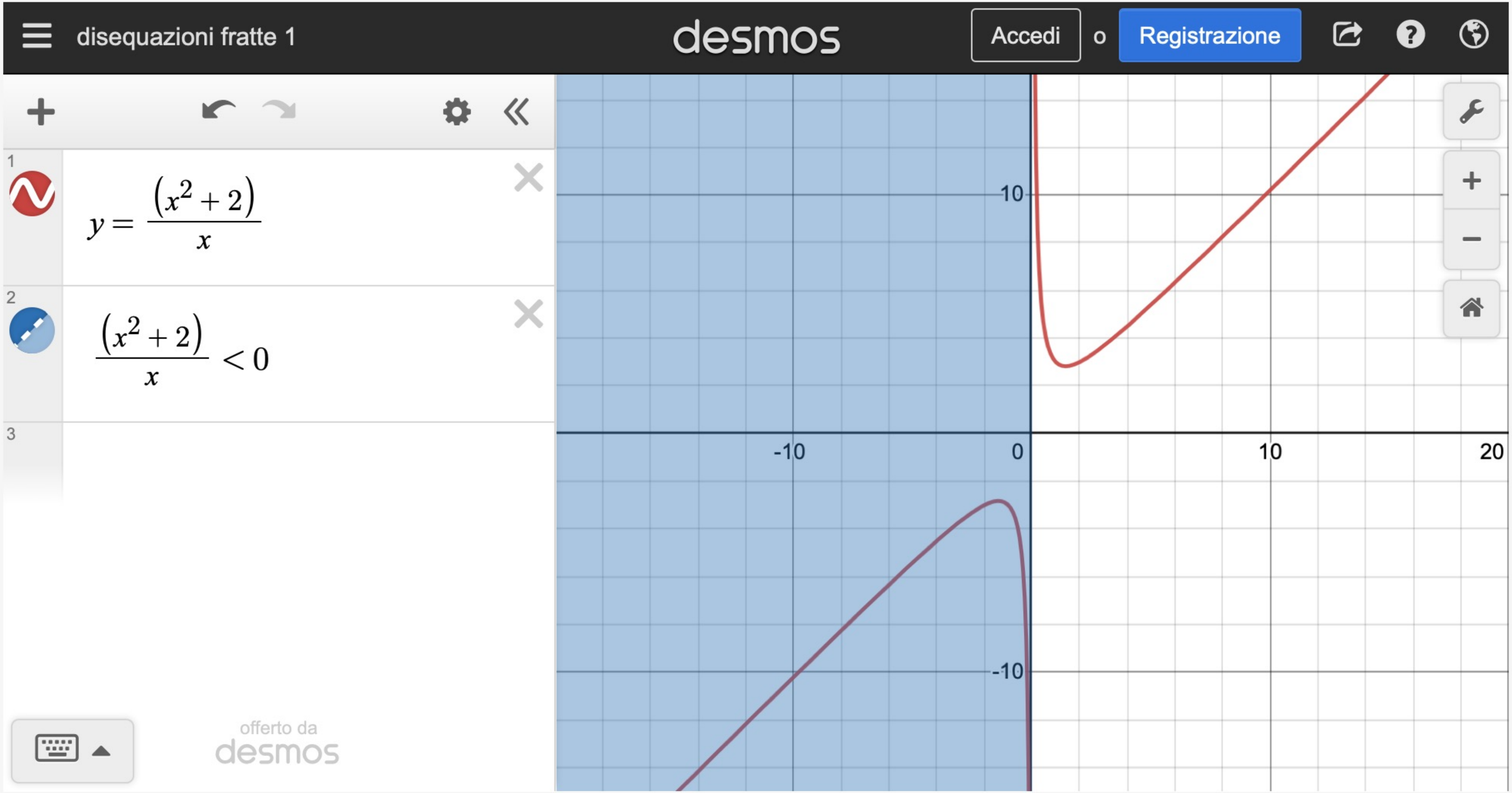
Parabola concava verso l'alto ed è sempre positiva

3) **Rappresento sulla retta orientata:**



4) **Soluzione S:** $x < 0$

La soluzione sarà data dai valori discordi perché ho risolto una disequazione Fratta con il segno negativo < 0



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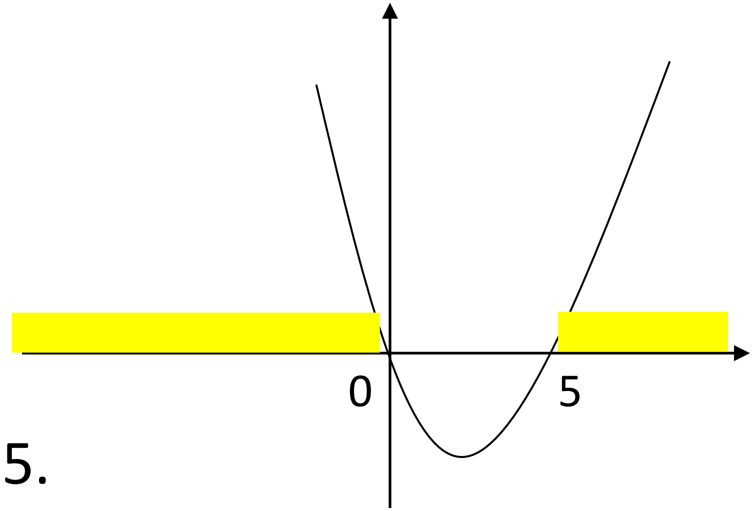
Esempio 2

$$\frac{x^2 - 5x}{x - 6} \geq 0$$

1) Pongo $x^2 - 5x \geq 0$

Pongo $x - 6 > 0$

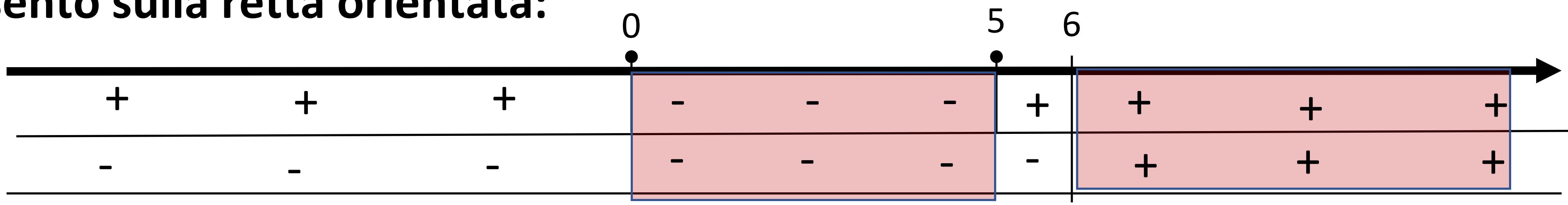


2) Risolvo singolarmente :

$N > 0.$ $x^2 - 5x \geq 0$ → S1: $x \leq 0 \vee x \geq 5.$

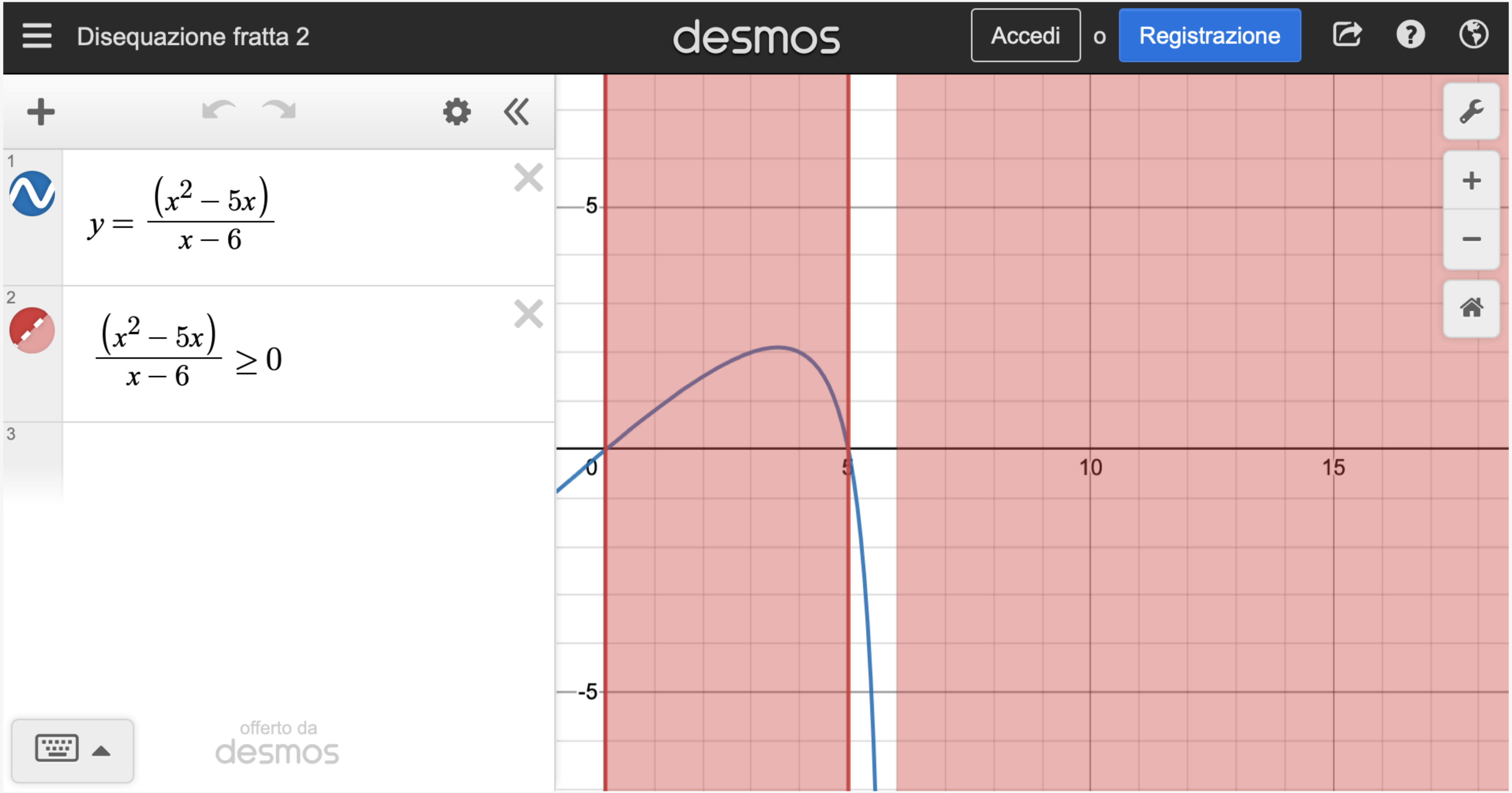
$D > 0.$ $x - 6 > 0$ → S2: $x > 6$

3) Rappresento sulla retta orientata:



4) Soluzione S: $0 \leq x \leq 5.$ V $x > 6$

La soluzione sarà data dai valori concordi perché ho risolto una disequazione Fratta con il segno positivo o uguale a zero ≥ 0



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Esercizi 3

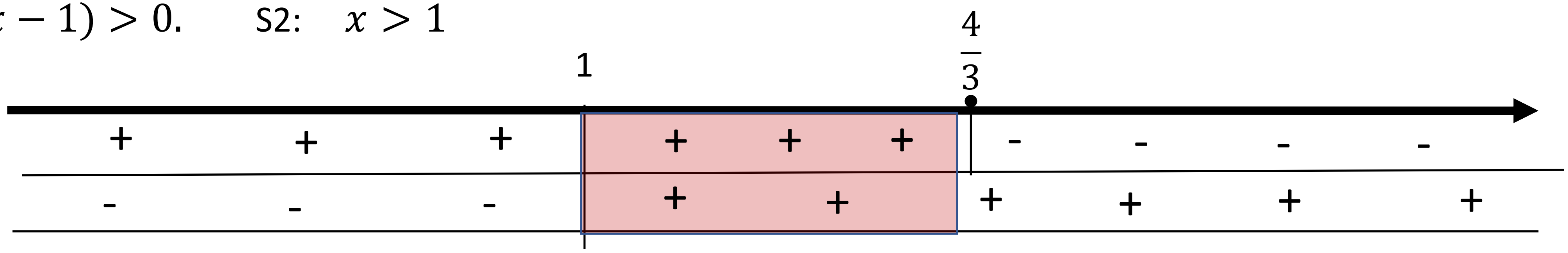
$$\frac{x}{2-2x} - 1 \geq \frac{1}{3x-3}$$

$$\frac{-x}{2x-2} - 1 - \frac{1}{3x-3} \geq 0$$

$$\frac{3x - 6(x-1) - 2}{2 \cdot 3 \cdot (x-1)} \geq 0 \qquad \frac{3x - 6x + 6 - 2}{6(x-1)} \geq 0 \qquad \frac{-3x+4}{6(x-1)} \geq 0$$

$N > 0.$ $\quad -3x + 4 \geq 0.$ $\quad$ S1: $\quad x \leq \frac{4}{3}.$

$D > 0.$ $\quad 6(x-1) > 0.$ $\quad$ S2: $\quad x > 1$



Soluzione $\frac{4}{3} \geq x > 1$

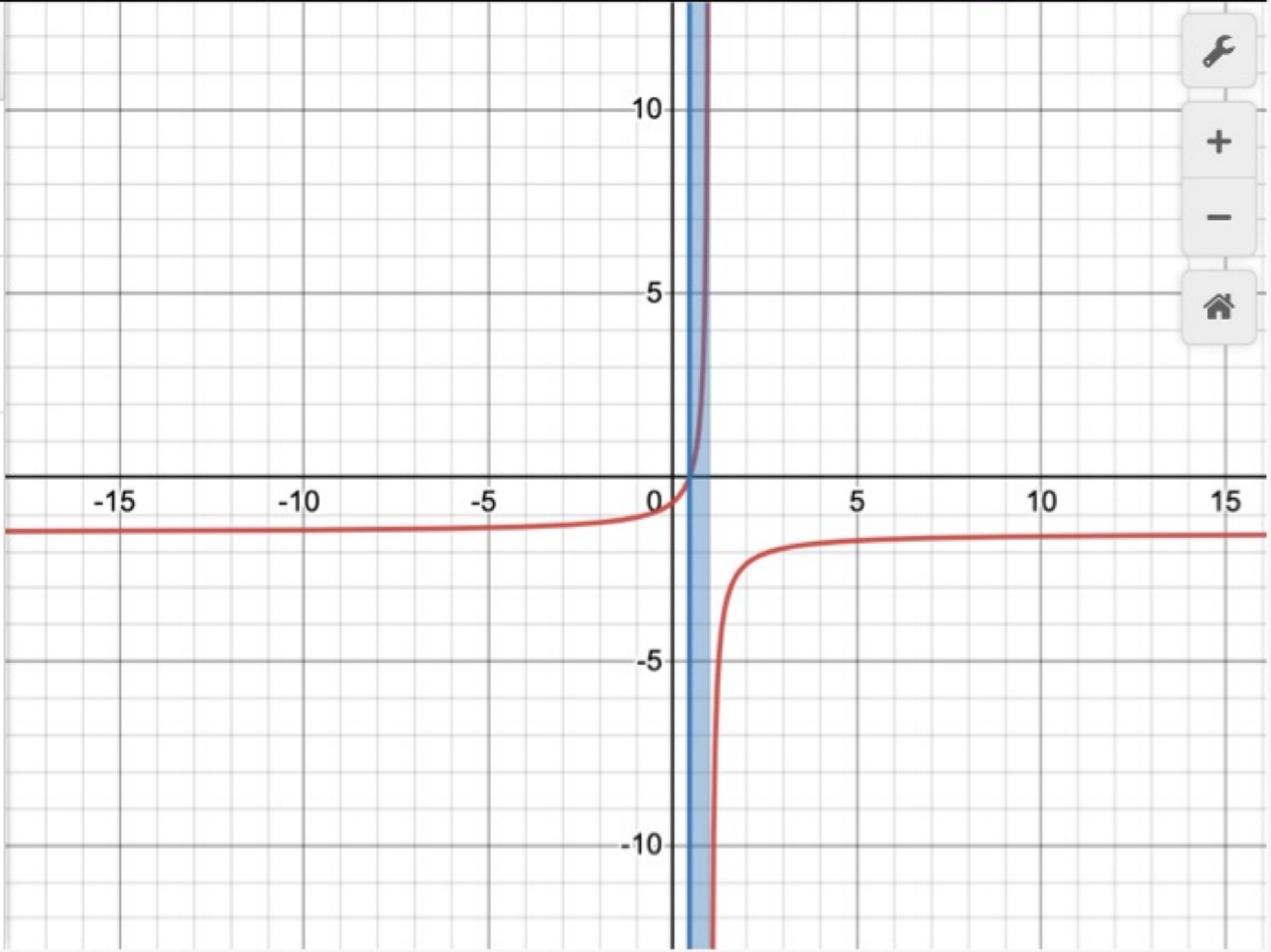
Disequazioni fratte 3 desmos Accedi o Registrazione

1 $y = \frac{x}{2-2x} - 1 - \frac{1}{3x-3}$

2 $\frac{x}{2-2x} - 1 - \frac{1}{3x-3} \geq 0$

3

offerto da desmos



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Esercizi 4

C.E. $\forall x \in R - \{-2\}$

$$\frac{x^2 - 7x}{x + 2} \geq 0$$

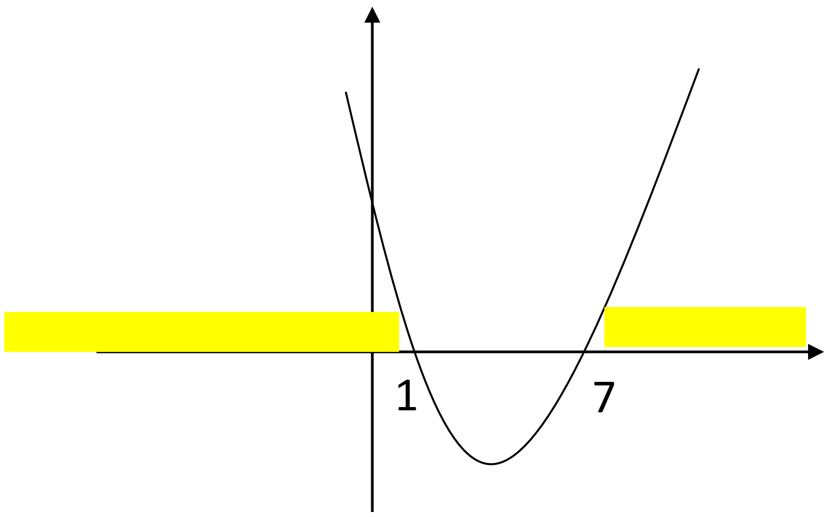
Numeratore

1) $x^2 - 7x \geq 0$

$x(x - 7) = 0$

$x_1 = 0$

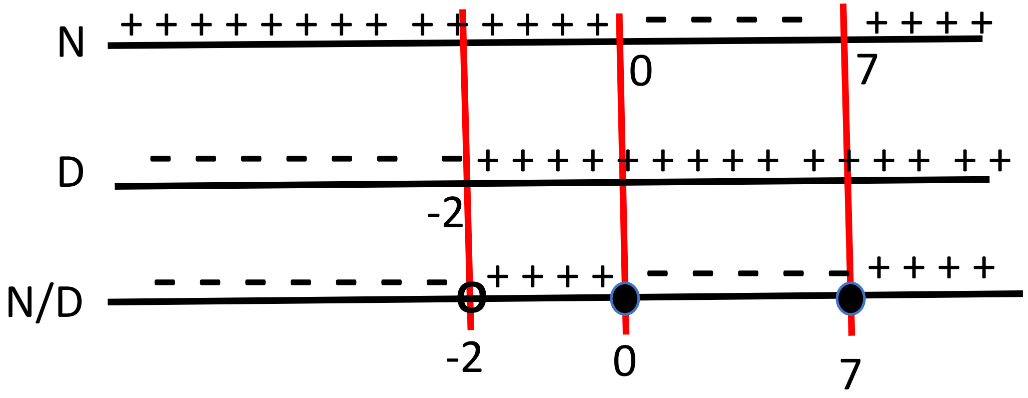
$x_2 = 7$



Denominatore

2) $x + 2 > 0$

$x > -2$



Soluzioni:

$-2 < x \leq 0$ V. $x \geq 7$

disequazioni fratte 4 desmos Accedi o Registrazione

1. $y=\frac{\left(x^2-7x\right)}{x+2}$

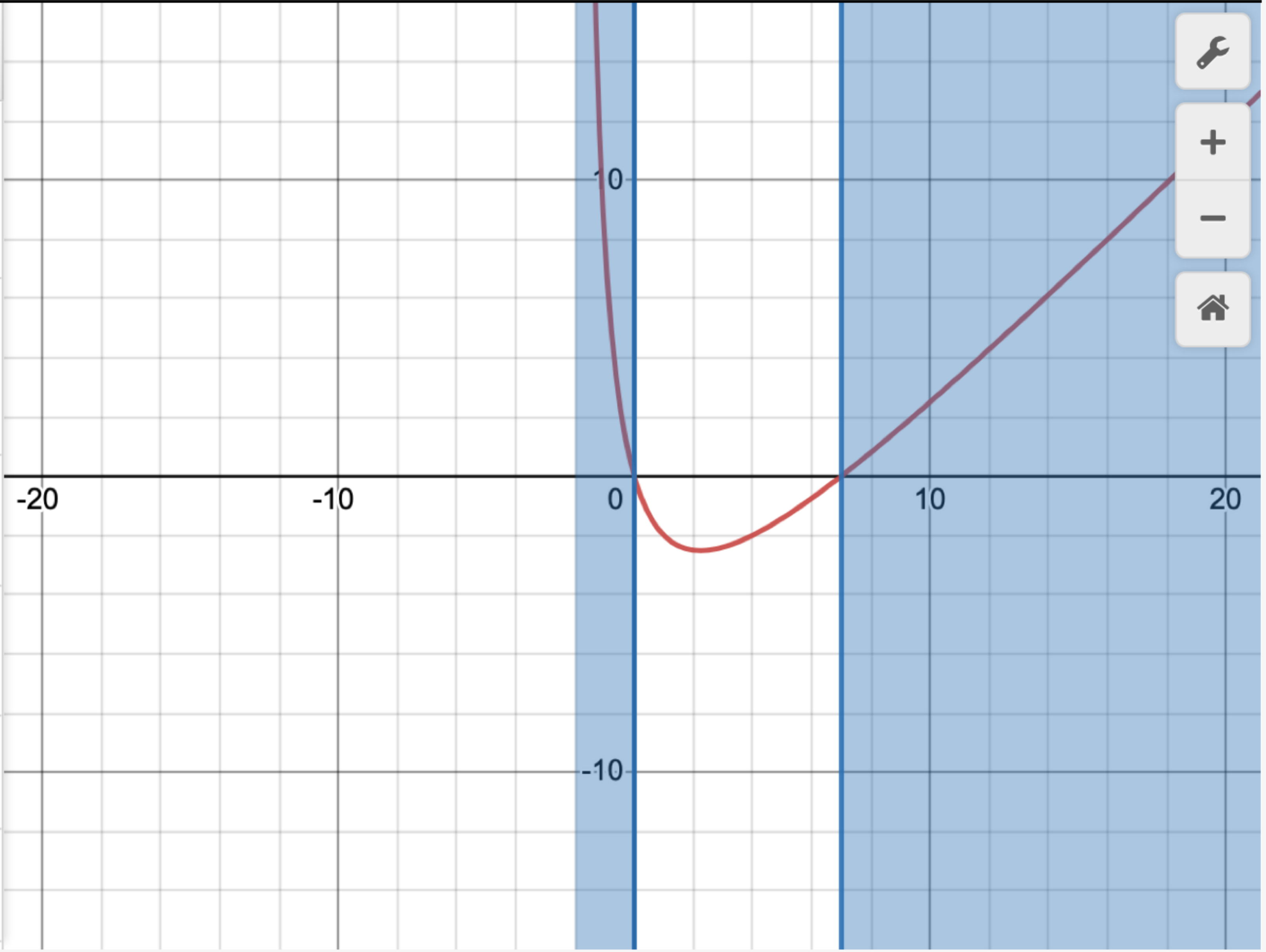
2. $\frac{\left(x^2-7x\right)}{x+2}\geq 0$

3. $x^2-7x\geq 0$

4. $y=x^2-7x$

5. $x+2>0$

6. $x+2$



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Mathematica

Esercizi 5

$$\frac{4x^2 - 3x}{x - 1} \geq 0$$

Esercizi 6

C.E. $\forall x \in R - \{3\}$

$$\frac{2x^2+3x-2}{x-3} \leq 0$$

1) $2x^2+3x-2 \geq 0$

$$2x^2+3x-2=0$$

$$\Delta = 3^2 - 4(2)(-2) = 25$$

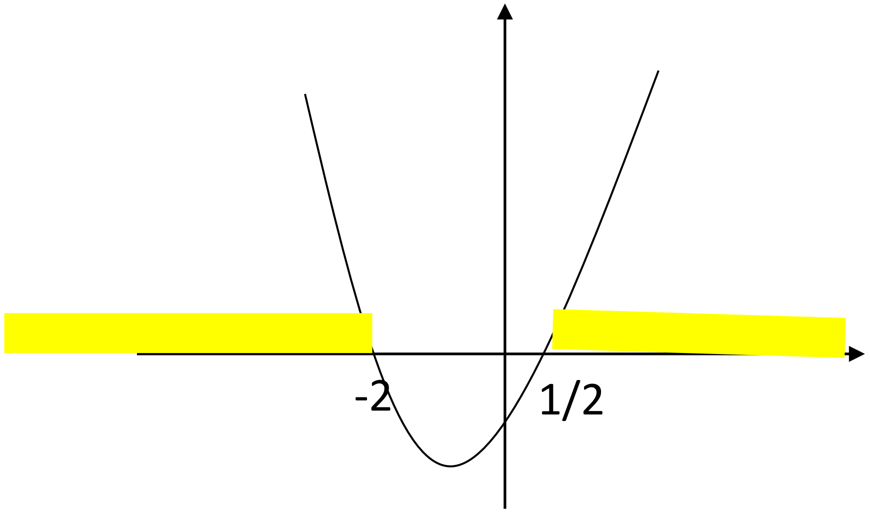
$$x_{1/2} = \frac{-3 \pm \sqrt{25}}{2 \cdot 2} = \frac{+3 \pm 5}{4} =$$

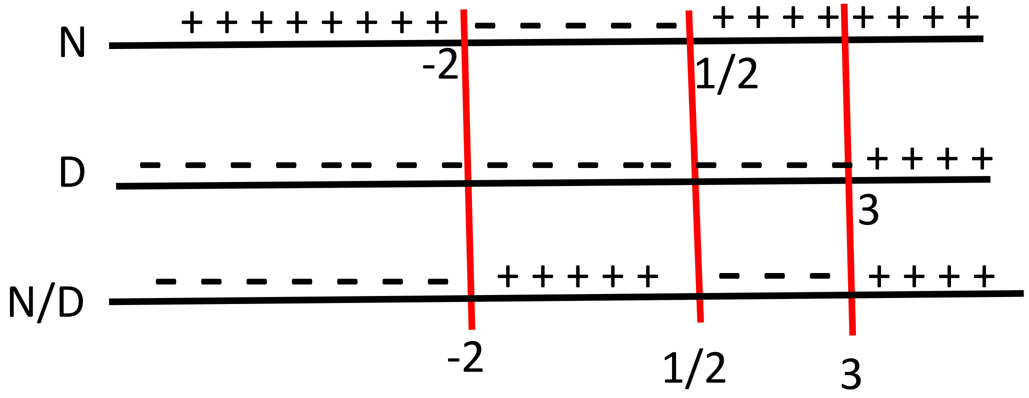
$$x_1 = \frac{-3+5}{4} = \frac{2}{4} = \frac{1}{2} \qquad x_1 = \frac{1}{2}$$

$$x_2 = \frac{-3-5}{4} = \frac{-8}{4} = -2 \qquad x_2 = -2$$

2) $x-3>0$

$x>3$





Soluzioni:

$$\frac{1}{2} \leq x \leq 3. \quad \text{V.} \quad x \leq -2$$

disequazioni fratte 6

desmos

Accedi o Registrazione

1. $$\frac{(2x^2+3x-2)}{x-3} \le 0$$

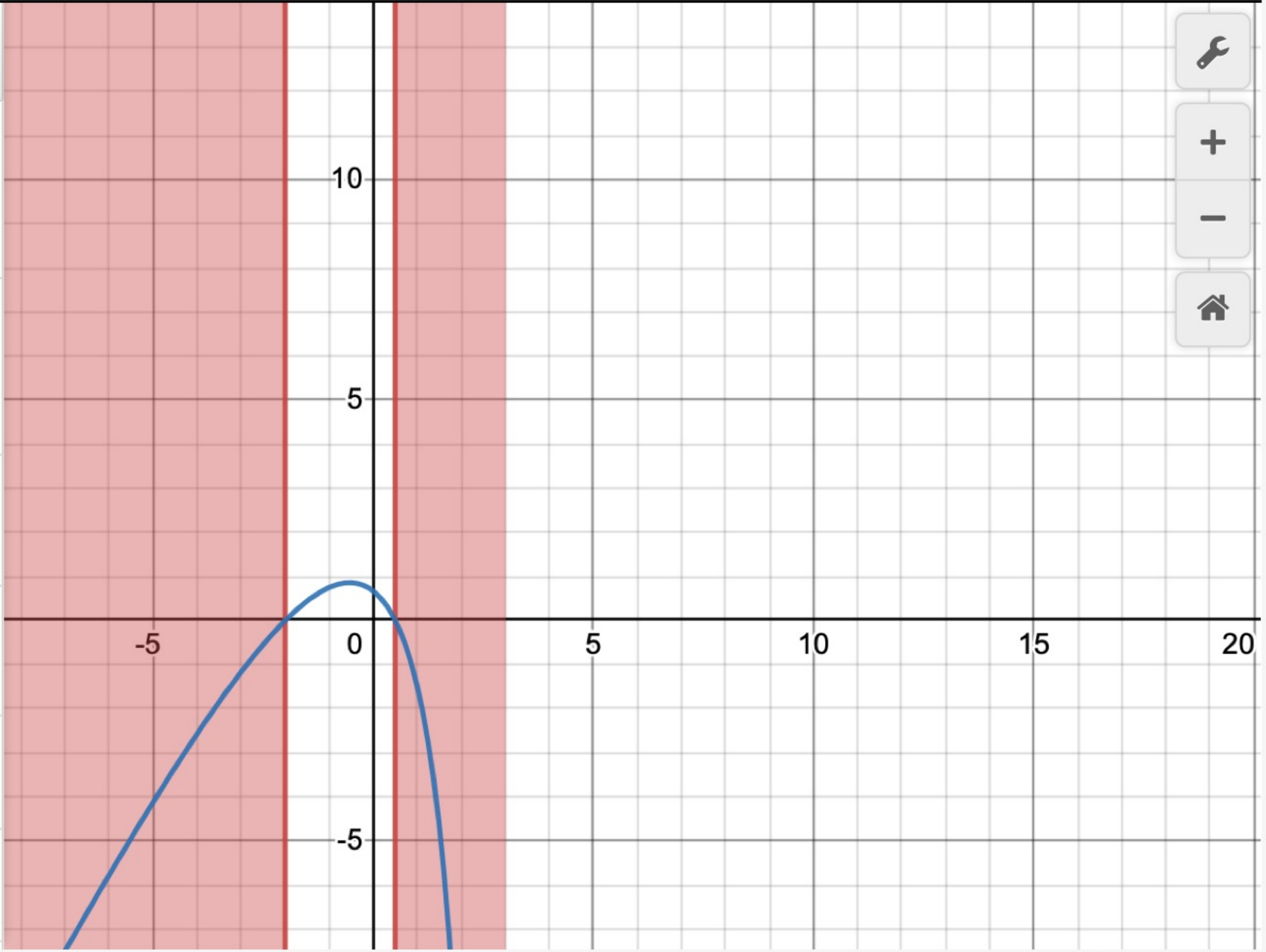
2. $$y = \frac{(2x^2+3x-2)}{x-3}$$

3. $$y = 2x^2+3x-2$$

4. $$2x^2+3x-2 \ge 0$$

5. $$y = x-3$$

6. $$3 > 0$$



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Esercizi

244 $\frac{1}{x-2} > -2$ $\left[x < \frac{3}{2} \vee x > 2\right]$

245 $\frac{2}{x+3} < 1$ $[x < -3 \vee x > -1]$

246 $-\frac{1}{x-1} \leq 2$ $\left[x \leq \frac{1}{2} \vee x > 1\right]$

247 $\frac{1}{x} > \frac{1}{x-2}$ $[0 < x < 2]$

248 $\frac{1}{2x-2} - \frac{1}{1-x} \geq \frac{2}{3x-3}$ $[x > 1]$

249 $-\frac{1}{2x+4} < \frac{x-1}{x+2}$ $\left[x < -2 \vee x > \frac{1}{2}\right]$

250 $\frac{4}{x+2} \geq 3 - x$ $[-2 < x \leq -1 \vee x \geq 2]$

251 $\frac{x}{x+1} \leq -\frac{2}{x}$ $[-1 < x < 0]$

Esercizi

Risolvi le seguenti disequazioni.

281 $\dfrac{x}{x^2-16} \leq 0$ $[x < -4 \vee 0 \leq x < 4]$

282 $\dfrac{3-x}{x^2-4} < 0$ $[-2 < x < 2 \vee x > 3]$

283 $\dfrac{5-x}{x^2-2x-4} \geq 0$ $[x < 1-\sqrt{5} \vee 1+\sqrt{5} < x \leq 5]$

284 $\dfrac{2x^2+5x-7}{2x} \geq 0$ $\left[-\dfrac{7}{2} \leq x < 0 \vee x \geq 1\right]$

285 $\dfrac{x^2-3x}{x^2-4} > 0$ $[x < -2 \vee 0 < x < 2 \vee x > 3]$

291 $\dfrac{2-x}{x^2-1} < 0$ $[-1 < x < 1 \vee x > 2]$

292 $\dfrac{x^2+5x-6}{x} \geq 0$ $[-6 \leq x < 0 \vee x \geq 1]$

293 $\dfrac{x}{x^2-25} \leq 0$ $[x < -5 \vee 0 \leq x < 5]$

294 $\dfrac{x^2}{x^2-4} \geq 0$ $[x = 0 \vee x < -2 \vee x > 2]$

295 $\dfrac{16-x^2}{x-3} < 0$ $[-4 < x < 3 \vee x > 4]$

Esercizi

286 $\dfrac{x-3x^2}{2x^2+3x-5} \geq 0$ $\left[-\dfrac{5}{2} < x \leq 0 \vee \dfrac{1}{3} \leq x < 1\right]$

287 $\dfrac{x^2-x-12}{x} \leq 0$ $[x \leq -3 \vee 0 < x \leq 4]$

288 $\dfrac{x^2-3x+5}{x^2-9} \leq 0$ $[-3 < x < 3]$ o

289 $\dfrac{2x-x^2-3}{2x^2-x-1} \leq 0$ $\left[x < -\dfrac{1}{2} \vee x > 1\right]$ o

290 $\dfrac{2-x}{x^2-2x-5} \geq 0$ $[x < 1-\sqrt{6} \vee 2 \leq x < 1+\sqrt{6}]$ o

296 $\dfrac{x-3}{-x^2+x+6} \leq 0$ $[x > -2 \wedge x \neq 3]$

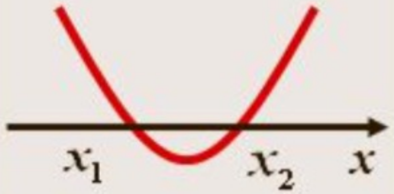
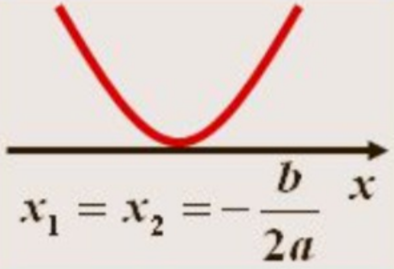
297 $\dfrac{x^2-1}{x^2-2x-6} \geq 0$ $[x < 1-\sqrt{7} \vee -1 \leq x \leq 1 \vee x > 1+\sqrt{7}]$

298 $\dfrac{x^2-4(x+1)^2}{3x-x^2} \leq 0$ $\left[-2 \leq x \leq -\dfrac{2}{3} \vee 0 < x < 3\right]$

299 $\dfrac{(2x+1)^2-x^2}{2x-x^2-2} > 0$ $\left[-1 < x < -\dfrac{1}{3}\right]$

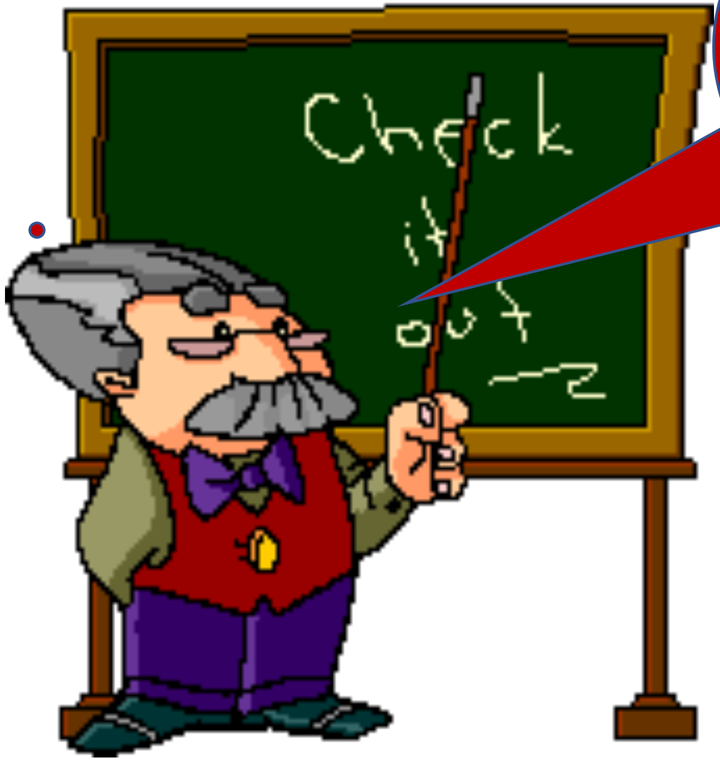
300 $\dfrac{3-6x}{x^2-5} \geq 0$ $\left[x < -\sqrt{5} \vee \dfrac{1}{2} \leq x < \sqrt{5}\right]$

Ricordiamo

$a > 0$	parabola	$ax^2 + bx + c > 0$	$ax^2 + bx + c < 0$
$\Delta > 0$		$x < x_1 \vee x > x_2$	$x_1 < x < x_2$
$\Delta = 0$		$\forall x \in \Re - \left\{ -\frac{b}{2a} \right\}$	$\nexists\, x \in \Re$
$\Delta < 0$		$\forall x \in \Re$	$\nexists\, x \in \Re$

Mathematica ML

Speriamo bene !!!



Per ora fermiamoci qua.